

Dynamics of group actions: a conference in the honor of Yves Benoist

List of abstracts

Bekka, Bachir (Université Rennes 1)

“Characters of infinite groups and invariant measures on homogeneous spaces”

Let G be a countable infinite group. Unless G is virtually abelian, a description of the unitary dual of G (that is, the equivalence classes of irreducible unitary representations of G) is hopeless. A sensible substitute for the unitary dual is the set $E(G)$ of (finite) characters of G . The characters of G , which are central positive definite functions on G , can be viewed as generalized invariant random subgroups (IRS) of G . We will give an overview of recent results about the description of $E(G)$ for some classes of groups G . An emphasis will be put on the crucial rôle of the classification of invariant measures on homogeneous spaces in these results.

Bray, Harrison (University of Michigan)

“Geometry and dynamics of the Benoist 3-manifolds”

In Benoist’s first convexes divisibles paper, he proved a strong dichotomy for convex domains which admit cocompact actions by discrete groups of projective transformations: the theorem says strict convexity of the domain coincides with delta-hyperbolicity of the abstract group. These properties also correspond to uniform hyperbolicity of the geodesic flow of the quotient. Three convexes divisibles papers later, Benoist introduced a class of examples in dimension 3 which are nonstrictly convex, but still have some hyperbolicity properties resembling rank one manifolds. In this talk, we present results on the dynamics of the geodesic flow in this nonuniformly hyperbolic setting, where the geometry is neither smooth nor Riemannian, but benefits from a powerful structure theorem of Benoist.

Breuillard, Emmanuel (University of Cambridge)

“The joint spectrum”

The notion of joint spectral radius of a set S of matrices was introduced by Rota and Strang in the 60’s and encodes the maximum asymptotic rate of spatial growth of a product of elements from S . It is intimately related to the maximal growth of eigenvalues of products of elements from S by theorems of Berger-Wang and Bochi. In this talk I will present a multi-dimensional version of this notion, where one looks at the full vector of eigenvalues leading naturally to the notion of joint spectrum of S . This is a compact subset of the Weyl chamber that can sometimes be explicitly computed and has connections with the asymptotic shape of large balls in Cayley graphs of Lie groups, with Lyapunov exponents for stationary processes, large deviations for random matrix products and with ergodic optimization.

Joint work with Çağrı Sert.

Brown, Aaron (University of Chicago)

“Stationary measures and orbit closures for actions on surfaces”

Inspired by the work of Benoist-Quint, we consider a finitely generated group of (volume-preserving) diffeomorphisms of a surface S and consider the questions of invariant and stationary

measures and orbit closures for the action. Assuming the action satisfies a certain dynamical criterion, we show all measures are volume or are finite and conclude every orbit is finite or dense.

Burger, Marc (ETH Zürich)

“Maximal representations over real fields, Λ -buildings and currents”

Given a maximal representation ρ of a lattice $\Gamma < \mathrm{PSL}(2, \mathbb{R})$ into $\mathrm{SP}(2n, \mathbb{F})$ where \mathbb{F} is a real closed field and $v : \mathbb{F}^\times \rightarrow \Lambda$ a non-archimedean valuation into an ordered \mathbb{Q} -vector space Λ , Γ acts on the associated Λ -building in the sense of C. Bennett $\mathcal{B}_n(\Lambda)$, and every $\gamma \in \Gamma$ has a well-defined translation length in Λ . When $\Lambda \subset \mathbb{R}$, we present an explicit construction of a geodesic current encoding the above translation lengths. For instance, we show that if the image under v of the trace field of ρ is discrete, this current is a finite multicurve. If the support \mathcal{L} of the current is a geodesic lamination, we show that the set of complementary regions of \mathcal{L} acquires the structure of a 0-hyperbolic Λ -metric space that embeds equivariantly and isometrically into $\mathcal{B}_n(\Lambda)$.

Joint work with A. Iozzi, A. Parreau and B. Pozzetti.

Eskin, Alex (University of Chicago)

“Exponential drift and character varieties”

Let G be a compact Lie group, and let S be a surface group. The character variety is the set of homomorphisms from S to G , modulo conjugation. Since the mapping class group is the outer automorphism group of S , it acts on the character variety by precomposition. It is natural to ask about stationary measures and orbit closures for this action.

The breakthrough work of Benoist and Quint and subsequent developments gives a new set of tools for attacking this problem. I will give a progress report and describe the difficulties in the general case.

Filip, Simion (Harvard University & IAS)

“Dynamics on K3 surfaces: homogeneous and inhomogeneous”

K3 surfaces are defined by algebraic equations and have moduli spaces that are homogeneous for appropriate Lie groups. At the same time, K3s admit dynamically interesting automorphisms and geometric structures which are far from homogeneous. I will discuss the interplay between the homogeneous structures on the moduli spaces and the geometry and dynamics of K3 automorphisms.

Joint work with Valentino Tosatti.

Foulon, Patrick (CIRM)

“Geodesic behaviour for Finsler metrics of constant flag curvature”

I hope to talk about some dynamical properties of Finsler spaces of constant curvature (in a weak sense). With R. Bryant, S. Ivanov, V. Matveev and W. Ziller.

We show that the geodesic flow of any Finsler metric of constant positive flag curvature is Liouville integrable, and we prove that, in dimension 2, the geodesic flow is conjugate to one of the Katok metrics.

The second part is based on a joint paper with I.Kim.

We study the Sinai-Ruelle-Bowen measure of convex projective structures. We observe that its entropy varies continuously and we also get some information about the geometry of the boundary at infinity.

Gorodnik, Alexander (University of Zürich & University of Bristol)
“Central limit theorems in dynamics and number theory”

We discuss statistical properties of orbits for group actions on homogeneous spaces. It turns out that this problem is also related to behavior of certain arithmetic counting functions. We shall show that in many cases averaging operators for group actions and arithmetic counting functions exhibit random behavior.

Guéritaud, François (CNRS & Université de Lille)
“Uniform Lipschitz extension in bounded curvature”

We will prove the following: in the hyperbolic plane \mathbb{H} , for every $0 < C < 1$ there exists $C < C' < 1$ such that every C -Lipschitz map from a subset of \mathbb{H} to \mathbb{H} has a C' -Lipschitz extension to \mathbb{H} . This will include a survey of results (Kirszbraun, Valentine, Lang-Schroeder...) on the Lipschitz extension problem in spaces with curvature bounds.

Hamenstädt, Ursula (Rheinische Friedrich-Wilhelms-Universität Bonn)
“Random walks, boundary actions and proper actions on L^p -spaces”

We discuss a general method to use random walks to obtain information on actions of groups on horoboundaries of spaces they act isometrically. The main goal is to construct proper actions on L^p -spaces. We show that such actions exist for the mapping class group of a surface of finite type.

Kobayashi, Toshiyuki (University of Tokyo)
“Regular representations on homogeneous spaces”

I plan to discuss some basic questions about regular representations on X acted algebraically by real reductive groups G .

1. (function spaces) Does the group G have a “good control” on the space $C(X)$ of function on X ?
2. (L^2 theory) What can we say about “spectrum” for $L^2(X)$?

We highlight “multiplicities” and “temperedness” for these questions, and give their geometric criteria.

The second topic is a joint work with Yves Benoist.

If time permits, I will mention some applications to branching problems for restriction of infinite-dimensional representations.

Labourie, François (Université de Nice)
“Triangles, flag manifolds and surface groups”

Associated to the choice of a $SL(2, \mathbb{R})$ in a semi simple Lie group, one can define “triangles” in the corresponding flag manifolds. I will explain how a notion of path of triangles and their deformations allows to describe a Morse type Lemma.

This notion plays a central role in our recent proof with Kahn and Mozes of the existence of surface groups in (some) higher rank lattices.

Lim, Seonhee (Seoul National University)

“Hausdorff dimension in inhomogeneous Diophantine approximation”

In this talk, we will present a series of results on the Hausdorff dimension of badly approximable vectors and matrices in inhomogeneous Diophantine approximation. If a lattice has zero escape of mass on average, then the Hausdorff dimension of ϵ -badly approximable translates of that lattice does not have full Hausdorff dimension [LSS]. We will explain the dynamics and entropy bound involved in the proof. For real numbers, the corresponding (2-dimensional) lattice is singular on average iff there exists some ϵ such that the dimension of ϵ -bad vectors is full [BKLR]. We will finish with recent progress where we extend the latter result to higher dimensions.

The talk is based on joint work with Uri Shapira–Nicolas de Saxcé, Yann Bugeaud–Donghan Kim–Michal Rams, and Wooyeon Kim.

Lindenstrauss, Elon (The Hebrew University of Jerusalem)

“On measures invariant under higher rank abelian actions in positive characteristic”

The rigidity properties of group actions on arithmetic quotients of algebraic groups defined over a global field of positive characteristic ($\mathbb{F}_q(t)$ or a finite extension) are less well understood than their zero characteristic counterpart. There are inherent reasons for that — in positive characteristic the additive group of a local field has a huge number of closed subgroups, polynomials may have small image, and there is no analog to the rational numbers - no “smallest” global field. For these reasons, there is no satisfying analogue to Rudolph’s theorem in this context.

I will present joint work with Einsiedler and Mohammadi giving a measure classification result for measures invariant under the full diagonal groups on some arithmetic quotients over field of positive characteristic.

Nevo, Amos (Technion – Israel Institute of Technology)

“Entropy theory beyond amenable groups”

In recent years, the classical theory of entropy for a measure-preserving dynamical system has been revolutionized by the ground-breaking work of Lewis Bowen and of Brandon Seward. Two distinct definitions were proposed and developed for actions of general groups: sofic entropy (initiated by Bowen) and Rokhlin entropy (initiated by Seward). We will start with a very brief account of the latter, and then describe our own recently developed approach and the construction orbital Rokhlin entropy for probability-measure-preserving free actions of all countable groups. We will then formulate our main result, namely that orbital Rokhlin entropy satisfies a Shannon-McMillan-Breiman pointwise convergence theorem. We will demonstrate the dynamical significance of the entropy equipartition that this convergence theorem entails, and its relation to the boundary of the group in the case of actions of free non-Abelian groups.

Based on joint work with F. Pogorzelski (Leipzig University).

Salehi Golsefidy, Alireza (UC San Diego)

“Two new concepts for compact groups: Spectral independence and local randomness”

I will explain two new concepts for compact groups mentioned in the title. Their basic properties and their connections with the FAb property, quasi-randomness, and super-approximation will be outlined. I will present how these ideas help us show that a Borel probability measure m on the product of two compact open subgroups of two non-isomorphic simple analytic groups has spectral gap when its projection to each factor has.

Joint work with Amir Mohammadi and Keivan Mallahi-Karai.

Sambarino, Andrés (CNRS & Université Paris 6)

“Pressure forms on pure imaginary directions”

Anosov groups are a class of discrete subgroups of semi-simple algebraic groups analogue to what is known as *convex-co-compact groups* in negative curvature. Thermodynamical constructions equip the (regular points of the) moduli space of Anosov representations from Γ to G with natural positive semi-definite bi-linear forms, known as pressure forms. Determining whether such a pressure form is Riemannian requires non-trivial work.

The purpose of the lecture is to explain some geometrical meaning of these forms, via a higher rank version of a celebrated result for quasi-Fuchsian space by Bridgeman-Taylor and McMullen on the Hessian of Hausdorff dimension on pure bending directions.

This is work in collaboration with M. Bridgeman, B. Pozzetti and A. Wienhard.

Sanders, Andrew (Universität Heidelberg)

“Generalized opers and complex analogues of surface group representations into real Lie groups”

Given a compact Riemann surface X , a curious fact is the appearance of the space of holomorphic quadratic differentials on X simultaneously parameterizing complex projective structures on X , and Fuchsian representations of the fundamental group of X . This curiosity has deepened in the last 30 years, with the simultaneous parameterization of Hitchin representations and so-called opers via a tuple of holomorphic differentials on X . In this talk, we will explain how this apparent coincidence is an avatar of a deeper relationship between certain components of surface group representations into real Lie groups, and a generalized notion of opers of X .

All of the discussion presented is the result of joint work with Brian Collier.

de Saxcé, Nicolas (CNRS & Université Paris 13)

“Diophantine approximation on quadrics”

Following work of Kleinbock, Merrill, Fishman and Simmons, we shall study the diophantine exponent of a point x on a quadric hypersurface. In particular, we shall explain how the classical theorems of Roth on algebraic numbers, and of Kleibock and Margulis on extremality of non-degenerate submanifolds can be adapted to our setting.

Schapira, Barbara (Université Rennes 1)

“Amenability of covers of negatively curved manifolds”

Let M be a negatively curved manifold. If M is “strongly positively recurrent”, i.e. there is a critical gap between its entropy at infinity and its entropy, we show that a cover M' of M is amenable if and only if the critical exponents of M' and M coincide. The proof uses a construction of Patterson-Sullivan measures twisted by a representation.

This is a joint work with R. Dougall, R. Coulon and S.Tapie.

Ulcigrai, Corinna (University of Zürich & University of Bristol)

“Mixing time-changes of nilflows”

Nilflows (flows on nilmanifolds, quotients of nilpotent Lie groups by a lattice) are one of the fundamental examples of *parabolic* dynamical systems. The simplest example is given by Heisenberg nilflows on quotients of the Heisenberg group. The talk will concern the ergodic

properties, in particular *mixing*, of parabolic flows which are obtained by *time-changes* (or time-reparametrizations) of nilflows. It is well known that nilflows are never mixing (for the presence of a toral factor). On the contrary, we will show that mixing is *typical* among (a class of) time-changes and we describe the mechanism which produces it. Our main result in particular generalizes to any nilflows (of step at least 2) previous results by Avila-Forni and myself for Heisenberg nilflows and Ravotti for filiform nilflows.

Joint work with Artur Avila, Giovanni Forni and Davide Ravotti.

Wang, Zhiren (Pennsylvania State University)
Continuous rigidity of higher rank lattice actions on tori

A recent result of Brown, Rodriguez Hertz and myself states that smooth Anosov actions by higher rank lattices on tori and nilmanifolds are smoothly conjugate to affine actions under a lifting hypothesis. In this talk, we will explain why continuous action by higher rank lattices on tori and nilmanifolds are continuously semiconjugate to affine actions, without assuming either hyperbolicity or lifting.

This is a joint work with F. Rodriguez Hertz.

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